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EXPLOITING THE RELATIONSHIP BETWEEN MATHEMATICS AND THE REAL WORLD IN TEACHING THE CHAPTER "TRIGONOMETRIC RELATIONS IN RIGHT TRIANGLES" (MATHEMATICS 9)

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Abstract: Connecting mathematics to the real world in teaching mathematics in schools is an issue that is of interest in research today. This connection helps teachers create motivation, opportunities for them to practice math and such skills as communication and cooperation. With the growing importance of real-world connections in math class, teachers need to be conscious and able to integrate real-world problems into their lectures. In this paper, we study the exploitation of the relationship between mathematics and the real world when teaching the chapter "Trigonometric relations in right triangles" for grade 9.

Keywords: Math, real world, trigonometric relations in right triangles.

1. INTRODUCTION

Secondary school students are being adults, preparing to participate directly in labor production and social development. Therefore, it is very necessary to equip them with adaptive abilities while learning at school. In order to foster and improve these abilities, especially the ability to apply mathematics to the real world, then one of the important measures is to strengthen real-world problems in mathematics teaching and to know how to build as well as use these problems effectively. So far, there have been a number of studies on applied mathematics circuits in mathematics teaching at high school, such as (Anh, P., 2012), (Ngoc, B. H., 2003), (Vui, T., 2014), (Gokhan Karakoc & Cengiz Alacaci, 2015), etc. Trigonometric relations in right triangles at 9^{th} grade is an important topic in the secondary school mathematics curriculum, but the problem of exploiting the relationship between mathematics and the real world in teaching this content has not been properly cared for by teachers. This paper proposes some measures to exploit the relationship between mathematics and the real world when teaching the chapter "Trigonometric relations in right triangles" (Mathematics 9).

2. LITERATURE REVIEW

The application of trigonometric relations in right triangles to solve real-world problems in teaching mathematics at school has been interested in research by Vietnamese and international mathematical education researchers. Secondary school mathematics curriculum and textbooks often provide problems with practical relevance, especially those for the application of Trigonometric relations in right triangles. This is the basis for teachers to carry out research and exploit the relationship between mathematics and the real world. Exploiting the relationship between mathematics and practice is a research goal of many works such as (Anh, P., 2012), (Gokhan Karakoc & Cengiz Alacaci, 2015) or (Ngoc, B. H., 2003). These works offer advantages and difficulties in exploiting the relationship between mathematics and the real world.

3. METHODS AND RESULTS

3.1. Methods

The study and exploitation of the relationship between mathematics and the real world for students when teaching *Trigonometric relations in right triangles* has been done through studying materials, thereby grasping the concepts as well as the characteristics of connecting mathematics with the real world. Next is to study the topic of *Trigonometric relations in right triangles* in the following aspects: the position and role of this topic in exploiting the relationship between mathematics with the real world, adding necessary knowledge to help students have more tools to solve problems and understand the meaning of the topic for solving real-world problems.

3.2. The exploitation of the relationship between mathematics and the real world

The exploitation of the relationship between mathematics and the real world in teaching mathematics at school has been an ongoing research problem. Studies

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have shown the role of this association in developing mathematical competencies for students.

In the Programme for International Student Assessment (PISA), mathematical literacy has been referred to an individual's capacity to formulate, employ and interpret mathematics that describe what individuals do to connect the context of a problem with mathematics, thus solve the problem. In the new General Education Curriculum of Vietnam, mathematical education builds the connection between mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

According to *NCTM* (National Council of Teachers of Mathematics), school mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics. Thus, *NCTM* proposed connections standard as follows: Instructional programs from prekindergarten through grade 12 should enable all students to: - recognize and use connections among mathematical ideas; - understand how mathematical ideas interconnect and build on one another to produce a coherent whole; - recognize and apply mathematics in contexts outside of mathematics.

Hans Freudenthal in Gravemejier and Terwel (2010) argued that Realistic Mathematics Education is the correct way of teaching, and teaching mathematics should always start with reality and stay within reality. Research suggests that the use of real world connections (RWCs) improves students' motivation and helps their better understanding of mathematics.

When they learn mathematics in a real world context, it develops their ability to use mathematical skills in solving problems of adult life (Gainsburg, 2008; Özdemir & Üzel, 2011; Sorensen, 2006 in Gokhan Karakoc & Cengiz Alacaci, 2015).

3.3. The content of the chapter "Trigonometric relations in right triangles" (Mathematics 9)

Trigonometric relations in right triangles is a chapter in the current 9th grade mathematics textbook. The purpose of this chapter is to solve right triangles given two edges or one edge and an acute angle, specifically: - To form formulae on trigonometric ratios of an acute angle. The relationship between trigonometric ratios of two complementary angles; - To use a number table or a pocket calculator to find the trigonometric ratios of a given acute angle and vice versa, to find the acute angle when you know one of its trigonometric ratios; - Build the relationships between edges and angles in a right triangle. In addition to the relationships, the chapter also constructs the relationships between edges and altitudes, edges and their projections,... in a right triangle; - Apply the above contents to calculate the height of an object and the distance between two places in reality.

3.4. Some measures to exploit the relationship between mathematics and the real world when teaching the chapter "Trigonometric relations in right triangles" (Mathematics 9)

In this part, we mention the relationship between mathematics and the real world when teaching the chapter "Trigonometric relations in right triangles" (Mathematics 9).

In order to help students enhance the ability to apply Trigonometric relations in right triangles into the real world, in our opinion, the Mathematics 9 textbook need to add some concepts of the following angle as follows:

The angle of elevation: of an object is the angle through which an observer's eye must be raised from the horizontal line to see the object (see *Figure 1*).

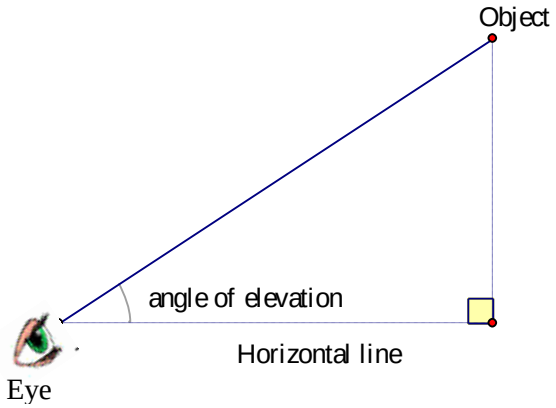

Figure 1

The angle of depression: of an object is the angle through which an observer's eye must be lowered from the horizontal line to see the object (see *Figure 2*).

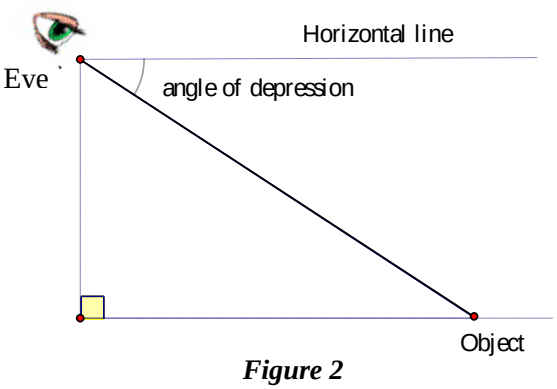

Figure 2

The addition of the concepts of these angles also helps students understand concepts of angles in the real world.

We next mention some measures to exploit the relationship between mathematics and the real world when teaching the chapter *Trigonometric relations in right triangles* (Mathematics 9).

3.4.1. Develop forms of mathematical problems about trigonometric relations in right triangles related to the real world

In order to diversify problems and gradually increase the difficult level of the tasks, we propose to develop the following mathematical forms:

Form 1: Applying Trigonometric relations in right triangles to measure the height of an object in real world.

Form 2: Applying Trigonometric relations in right triangles to measure lengths or distances in real world.

Form 3: Applying Trigonometric relations in right triangles to solve complex problems in real world.

Example 1: To see the peak A of a steep cliff, a man stood at the point P about 45m away from the cliff's foot and looked up at an angle of 25^0 from the horizontal line (see *Figure 3*). How high is the height of the cliff from the peak A to the ground?

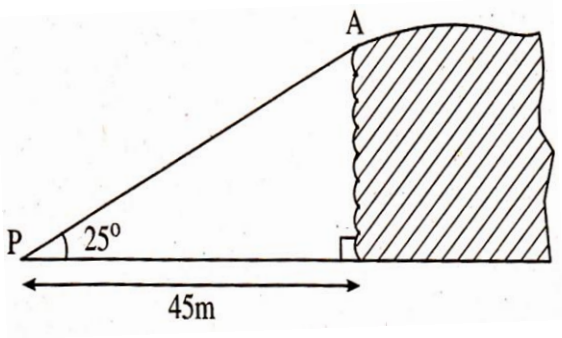

Figure 3

Hint: For this problem, students have the opportunity to apply knowledge of trigonometric relations in right triangles to calculate the height of an object in the real world, and understand the concept of "the angle of elevation" (in this situation, that is *angle 25^o from the horizontal line*).

Using the formula to calculate tangent in a right triangle, we get the result we need to find as follows: $45.\tan 25^o \approx 20{,}98$ (m).

Example 2: From the top of a sea lamp 38m above sea level, an island is seen through an angle of 30^o from the horizontal line of its foot (see *Figure 4*). What is the distance from the island to the foot of the lamp (at sea level)?

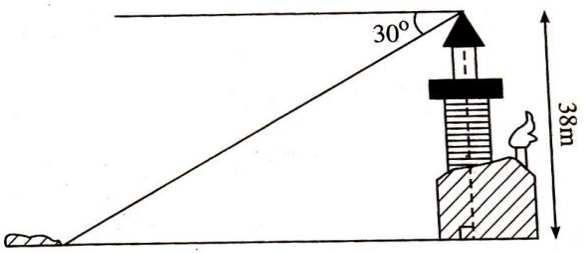

Figure 4

Hint: For this problem, students have the opportunity to apply knowledge of trigonometric relations in right triangles to calculate the distance in the real world, and understand the concept of "the angle of depression" (in this situation, that is *angle 30^o from the horizontal line of the foot of the lamp*).

Using the equal property of two alternate interior angles and the formula for calculating cotangent in a right triangle, we get the result we need to find as follows: $38.\cot 30^o = 65{,}82$ (m).

Alternatively, students can use the property of two complementary angles and the formula for calculating tangent in a right triangle, we get the result we need to find as follows: $38.\cot 30^o = 65{,}82$ (m).

Example 3: At two points 20 metres apart and on the same side of a tree, the angles of elevation of the top of the tree are 21^o and 16^o (see *Figure 5*). Find the height of the tree, correct to the nearest metre.

***Solution*:**

Start by drawing a diagram to depict the situation.

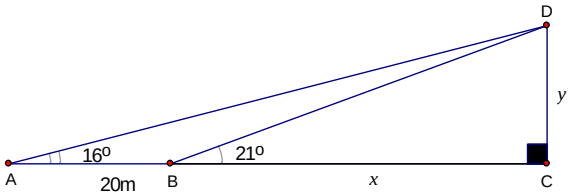

Figure 5

Next, indicate the relevant information, for example angles and distances in the diagram above.

To find the height, denoted by y, of the tree, we need to find the distance, denoted by x, as shown in the diagram above.

Since $BCD = 90^o$ then $\frac{y}{x} = \tan 21^o$ and $\frac{y}{x+20} = \tan 16^o$.

We have $y = x.\tan 21^o \approx 0{,}3839x$ (1).

Thus, we obtain $\frac{0,3839x}{x+20} = \tan 16^o$

This implies that $0,3839x = (x + 20)\tan 16^o$

$\approx (x + 20).0,2867$

$= 0,2867x + 5,734$

Therefore, $x = \frac{5,734}{0,0972} \approx 58,99$.

From (1), it follows that

y = 0,3839.58,99 ≈ 23m (to the nearest metre).

Hence, the height of the tree is 23 metres.

Note: We have used the following heuristics to solve the problem above: + Use diagram to depic the situation; + Make connection between the context of the problem and our knowledge of the trigonometric ratios; + Use equations to solve them.

3.4.2. Use real-world situations to motivate students to form mathematical concepts and consolidate knowledge of trigonometric relations in right triangles

In order to form and consolidate mathematical concepts on the basis of real-world situations, teachers can take the following steps:

Step 1: Motivating to form the mathematical concept. Teachers can use real-world situations to help students see the existence or effects of a kind of certain object.

Step 2: Stating the definition of the mathematical concept.

Step 3: Applying and consolidating the mathematical concept. Teachers can ask students to find images of concepts in real world or give some real-world exercises related to the concept for students to apply.

Example 4: When teaching the concepts of the trigonometric ratios of an acute angle in a right triangle, teachers can present some real-world situations to motivate students to see the significance of mathematical concepts what are going to be learned:

Situation 1: A pylon 7*m* high casts a shadow 4*m* long on the ground (see *Figure 6*). Calculate the angle (correct to minute) that the sun's ray creates to the ground.

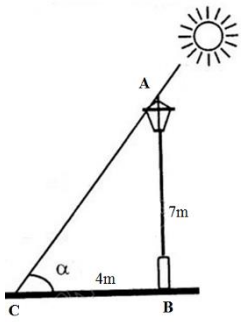

Figure 6

Situation 2: A ladder *AB* 6,7*m* long leans against a wall and makes an angle of 63° with the ground (see *Figure 7*). How high up the wall does the ladder reach? How far is the foot B of the ladder from the wall?

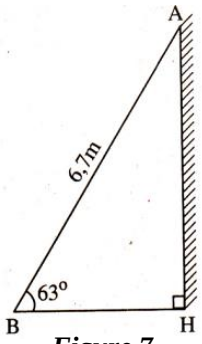

Figure 7

Situation 3: A section of a river is about 250*m* wide. A rowboat across the river was pushed obliquely 320*m* by water flow to cross the other bank (see *Figure 8*). Calculate the angle that water flow pushed the boat?

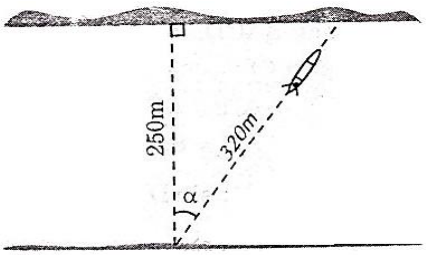

Figure 8

After the students has learned the concepts of trigonometric ratios, the teacher asks the students to address the situations described above as an option for students to use mathematical knowledge to solve real-world problems.

Similar to teaching mathematical theorems concerning the relations between the sides and altitudes or the sides and angles in a right triangle, teachers can also use the real-world situations to motivate students to learn mathematical theorems comes from a need arising in real world related to the above relations in a right triangle. Depending on the content of the unit, teachers may choose real world situations to motivate students to detect theorems.

To consolidate a theorem, teachers may ask students to solve some real world situations by using the theorem. These situations are presented in the form of realistic exercises. The system of exercises with real-world content should be constructed corresponding to all the

Trigonometric relations in right triangles given in the Mathematics 9 textbook.

3.4.3. Organizing extra-curricular mathematics activities whose contents relate to applications of Trigonometric relations in right triangles into the real world

In our opinion, the phases of conducting an extracurricular activity involving the application of knowledge of Trigonometric relations in right triangles to solve real-world problems include:

Preparation phase: Teachers select the topic of practice, define the practice plan, prepare equipments and tools (Gauge, tape measures, pocket calculators,...), assign practice placements, inspect, arrange of tools, materials.

Implementation phase: + Step 1: Warm-up; + Step 2: The teacher presents and models; + Step 3: Students repeat and explain; + Step 4: Practise independently.

Ending phase: The teacher analyzes the performance results compared to the intended purpose, answers the questions, and notes the errors that students have made, consolidates the knowledge through the content of the practise.

4. DISCUSSION AND CONCLUSION

Trigonometry in general, Trigonometric relations in right triangles in particular have many important applications in the real world. Thus, the three directions exploiting the relationship between mathematics and real world in teaching Trigonometric relations in right triangles in the 9^{th} grade class above have dual effects, that are both to help students see the role and significance of learnt knowledge and to train them the skill to connect mathematics with real world.

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