

## TRAINING MATHEMATIC TEACHERS IN ACCORDANCE WITH TEACHING TO INTEGRATED MATH AND SCIENCE THROUGH TEACHING INTEGRATION CONCEPT

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**Abstract:** At the beginning of the 20<sup>th</sup> century, integrated mathematics and science education has been an important topic and gained considerable attention in the last decades. The first part of this paper introduces the existing definitions, models and our own understanding of integrating mathematics and science. The following part demonstrates this understanding regarding the orientation of teachers' activities in an integrated classroom. In order to verify whether the knowledge of pre-service teachers is suitable for this orientation, we conducted an experiment with the concept of integral which is closely connected to physics. The results showed that students, despite having a solid comprehension of both subjects, faced difficulties in connecting both subjects' knowledge to solve problems in a physics context which requires using integral. It also revealed the potential of utilizing physics contexts to help learners have a better understanding of mathematical concepts. These results can possibly introduce a few proposals for the training program of pre-service teachers.

**Keywords:** Teacher education, mathematics and science integration, integral calculus.

### 1. INTRODUCTION

The demand to resolve problems in real life and science is the main driving force of the advancement of mathematics. It links mathematics with science closely together throughout history. Science creates problems thus motivate the development of new mathematics concepts and provides contexts in order to enrich the meanings of these concepts. Vice versa, mathematics has become an essential tool to solve problems in science. Therefore, this interdisciplinary connection must be taken into consideration in teaching mathematics and science. Integrated mathematics and science research in education is not new. In fact, according to Berlin and White (1999), it has begun since 1900s and gained considerable attention in the last decades. Education in Viet Nam in recent years has taken steps to follow the integrated orientation. However, research about integrated mathematics and science in education still lacks adequate attention. Furthermore, many studies in the world have identified some difficulties teachers can face in integrated mathematics and science courses. Therefore, in order to implement integrated mathematics and science teaching in the future, the training of pre-service teachers needs a beforehand approach. In this view, it is the right time to ask: *Is the traditional program of the University of Education still suitable for teaching integrated mathematics? What can we add in or change?*

*What kind of skills and knowledge will pre-service teachers need to equip themselves in order to teach integrated mathematics and science in the future?*

The first part of this paper introduces the existing definitions, models and difficulties that teachers can face in integrated mathematics and science teaching. We also present our own understanding of integrating mathematics and science and showed this understanding regarding the orientation of teachers' activities in an integrated classroom.

On the other hand, we believe integrated teachings may not be appropriate for every topic but instead, should be applied for the most suitable topics. Selecting the potential topics that have many connections between mathematics and science will be imperative in the process of designing integrated programs. If we had to choose only two areas of mathematics and science that are intertwined throughout history, it would be calculus and physics. Integral is perhaps one of the important concepts in calculus that have various applications in physics. In our previous study (Duc, N. M., 2017a), we concluded that integrated mathematics and physics is an appropriate way to teach integral in high school.

In order to verify whether the knowledge of pre-service mathematics teachers is suitable for this orientation, we conducted an experiment with the concept of integral. The results showed that students,

despite having a solid comprehension of both subjects, faced difficulties connecting both subjects' knowledge to solve problems in a physics context which requires them to use integration. It also revealed the potential of utilizing physics contexts to help learners have a better understanding of mathematical concepts. These results can possibly introduce a few proposals for the current training program of pre-service teachers.

## 2. LITERATURE REVIEW

### 2.1. *Integrated mathematics and science in education*

Integrated mathematics and science has had a long history. However, at first it focused on using mathematical tools in science fields yet ignoring an educational dimension. The following studies tied teaching with integrating mathematics and science and gave many understandings. As a result, multiple terminologies have been used to explain this integration such as: blended, connected, coordinated, fused, immersed, interactions, interdisciplinary or linked.

Berlin and White (1992) defined integrated mathematics and science as a blend of two fields from linking concepts, principles using the same methods. Frykholm and Glasson (2005) suggested that integration was an expansion of both mathematics and science from the connections between them. Frykholm and Glasson also believed that the difference in understanding "integrated" and "interdisciplinary" is from the integrity of disciplinary boundaries. While interdisciplinary teaching is considered to maintain the integrity of disciplinary boundaries through studying common contexts, definitions of 'integrated' may suggest that "science and mathematics can be blended seamlessly so that it is difficult to tell where the mathematics stops and the science begins" (p. 130). Nonetheless, Kiray (2012) suggested that these terms and understandings should be grouped as "integrated mathematics and science" and this is the term we will use frequently throughout this paper.

### 2.2. *Teachers' difficulties in integrated mathematics and science teaching*

Frykholm and Glasson (2005) suggested that teachers lack content knowledge from both fields and skills to integrate topics. Ball (1990) had proposed that teachers must possess knowledge in their field which was enriched by linking with other disciplines in order to do the integrated programs. Barnett and Hodson (2001) showed that teachers' skills and knowledge needed to be linked to the contexts in which these knowledges can be formed and developed. Frykholm and Glasson (2005) also encouraged teachers to understand contexts which

enable the connection between the meanings of science and mathematics knowledge.

### 2.3. *Integrated mathematics and science teaching models*

Berlin và White (1994) proposed the first model for integrated mathematics and science called BWISM (Berlin-White Integrated Science and Mathematics Model) based on five interactions: +) Math for math: M; +) Math - Science context: Ms; +) Math and Science: MS; +) Science - apply Math: Sm; +) Science for Science: S.

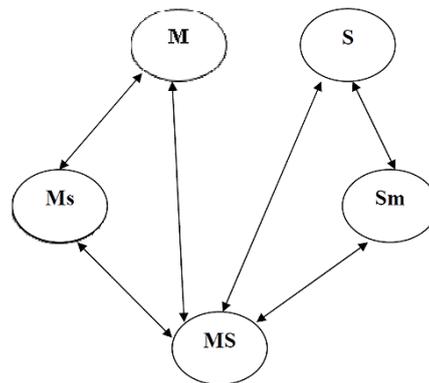


Figure 1. BWISM model

This model places integrated mathematics and science teaching (MS) as the center of the interactions, linking teaching mathematics in science contexts (MS - Ms) and applying mathematical tools in science fields (MS - Sm).

Lonning and DeFranco (1997: p. 213) also proposed a similar model when he divided the interactions between mathematics and science into five types: +) Independent mathematics: Includes concepts best taught in a purely mathematical context; +) Mathematics focus: Science concepts/ activities are in support of mathematics concepts; +) Balanced mathematics and science: Activities provide for integration of equally appropriate mathematics and science concepts/ activities; +) Science focus: Mathematics concepts/ activities are in support of science concepts; +) Independent science: Includes concepts best taught in a purely scientific context.

Based on the synthesis of integrated models in history, Hurley (2001: p. 263) proposed a model with five types of integration: +) Sequenced: Science and mathematics are planned and taught sequentially, with one preceding the other; +) Parallel: Science and mathematics are planned and taught simultaneously through parallel concepts; +) Partial: Science and mathematics are taught partially together and partially as separate disciplines in the same classes; +) Enhanced:

Either science or mathematics is the major discipline of instruction, with the other discipline apparent throughout the instruction; +) Total: Science and mathematics are taught together in intended equality.

### 3. METHODS AND RESULTS

#### 3.1. A balanced model for integrated mathematics and science teaching

We aimed towards a more balanced model that can benefit both teaching mathematics and science. Because of that, we have selected the two interactions MS - Ms and MS - Sm from the BWISM model. Following our model, we consider integrated mathematics and science teaching a process of utilizing connections and interactions between teaching mathematics and science. Specifically, teaching mathematics should include providing mathematical concepts, meanings and methods as tools applied in sciences. Conversely, science provides meaningful and relevant contexts to create concrete meanings to mathematical concepts and help learners to understand the concepts more. In addition, science brings situations that mathematical concepts were developed from just adding value to studying mathematics.

#### 3.2. Integrated mathematics and science in teacher training

Traditionally, in high school mathematics and science subjects are taught separately from each other. Because of that, mathematics teachers do not equip themselves with science knowledge and the connections between science contexts and mathematical concepts. Moreover, they are not required to teach mathematics in a way that help learners to apply it in science such as physics. Reforming mathematics teaching with the integrated mathematics and science orientation raises questions about the practice of teachers: - Do teachers have knowledge about the interdisciplinary connections between the content of mathematics and science? - Can teachers use science contexts appropriately in order to teach mathematical knowledge? - Do teachers link the content of mathematics with the purpose of helping learners to apply that knowledge in science problems?

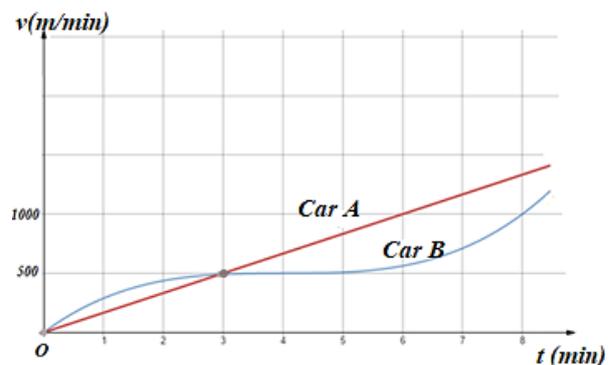
A “good teacher” in an integrated mathematics and science class obviously have to answer “yes” to all three questions above. In order to have teachers like that, we need a reform in the training of teachers in the University of Education and preparing appropriate materials for teachers. Integration in teaching orientation also requires research to investigate the difficulties teachers face when teaching integrated mathematics and science. From that, we can see which integrated skills and knowledge they

lack and what to prepare for pre-service teachers. This research will need both the contribution from mathematics and science educators. Within the scope of our paper, we will conduct an experiment to investigate the knowledge of mathematics pre-service teachers about the integral concept in connections to physics knowledge. This experiment aims to figure out whether mathematics pre-service teachers can use integral knowledge to solve physics problems. In addition, this experiment also highlights the potential of using physics to support the understanding of the concept of integral.

#### 3.3. Experiment with pre-service mathematics teachers about the concept of integral

We conducted our experiment with third-year mathematics students in the Ho Chi Minh City University of Education. 32 students were selected to solve three problems, the first two test the ability to use integral to solve problems in physics context. The last one indicates how interdisciplinary contexts between mathematics and physics can help students to understand the concept of integral deeply.

*Problem 1. Two cars starting at the same time and the same position begin to move straightforward in a line at the same direction. Their velocities are given by these two graphs:*



a. At the point, which car has gone the further distance? Explain your answer.

b. At the point, which car has gone the further distance? Explain your answer.

Analysis: From velocity-time graphs, we can find the velocity function of car A while car B cannot. One of the meanings of integral in physics which is taught in mathematics program is that the distance equals to the integral of velocity. In addition, integral also has a geometric meaning as the area under the curve. We want to check whether these students can compare two distances by comparing integrals and connecting with the

comparison of the areas under the velocity curves. Using the area to calculate or to compare quantities is an important technique in physics especially when the quantities are not always given by an algebraic representation.

Solving strategy

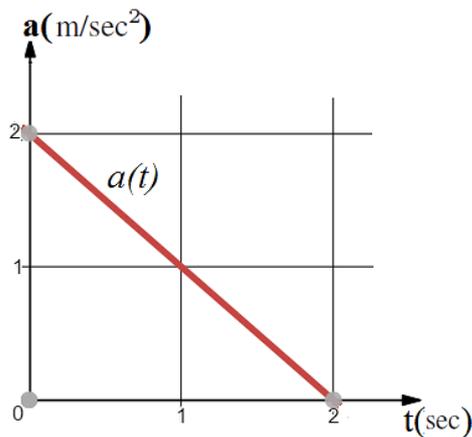
Integral-Area strategy ( $S_{Area}$ ): Comparing the distances by comparing the areas under the curve.

Integral-algebra strategy ( $S_{Algebra}$ ): Find the algebraic expression of the velocity and calculate the distance by integral then comparing the results.

Physics strategy [ $(S)_{Physics}$ ]: Comparing the distances through reasoning in physics context (larger velocity means further distance).

Results: In part a, most students use  $S_{Physics}$  to explain that the distance of car B is longer because its velocity is larger. However, in part b, when  $S_{Algebra}$  and  $S_{Physics}$  is unavailable to use, only 19% (6/32) of the students knows how to use  $S_{Area}$  to explain that car A has traveled further because its area under the curve is bigger. Most of the students cannot figure out the integral tool in this context, and even if they attempted to use integral, they weren't successful because they couldn't link the distance to the area under the curve.

Problem 2. The acceleration function of an object in the time interval  $[0; 2]$  is:  $a(t) = -t + 2$ . What is the change in its velocity from  $t = 0$  to  $t = 2$ ?



Analysis: Students are familiar with the Newton-Leibniz formula, where integral is calculated by using its antiderivative:  $\int_a^b f'(x)dx = f(b) - f(a) = \Delta f$ . In high school, students were taught that acceleration is the derivative of velocity:  $a(t) = v'(t)$ . Therefore, in this problem's context, the Newton-Leibniz will be:

$\Delta v = \int_a^b v'(t)dt = \int_a^b a(t)dt$ . This means the change in velocity is the integral of the acceleration function. We asked about the change in velocity at two points in time in this problem and can students utilize the knowledge of integral and physics to solve them?

Solving strategy:

Integral strategy ( $S_{Integral}$ ):

The change in velocity is:

$$\Delta v = \int_a^b a(t)dt = \int_0^2 (-t + 2)dt = 2(m/s).$$

Calculating integral by area strategy ( $S_{Area}$ ):

Because the acceleration function is expressed through a graph so we can calculate the integral by the area under the curve.

Results: Only 43.75% (14/32) students manage to use integral to find the change in velocity. In those students, 78.75% (11/14) students used  $S_{Integral}$  and 21.34% used  $S_{Area}$ .

The remaining students solved this by doing different physics calculation but did not succeeded. (There is no current formula in physics textbook for movement that has changing acceleration.

Problem 3. Work A done by constant force F moves an object by distance s (same direction with the force) is calculated by:  $A = F \cdot s$

In the case that the force changes through the function F(s). Grade 10 physics textbook proposed a method for calculating work:

"For a variable force, we can divide the distance travelled into small parts  $\Delta s$ , so we can consider the force on that part to be constant. The work done on by force on these small distances is calculated by  $\Delta A = F \cdot \Delta s$ . The total work done by the force is approximated by adding all that elementary work above".

An object moved by a decreasing force expressed by  $F(s) = 5 - \frac{1}{30}s^2$  (force is in the same direction of the movement). Based on this method, approximate the work done by the force when the object is moved from the starting point  $s = 0$  to  $s = 3$ .

Proposed a mathematical method to accurately calculate the work. Explain your proposal.

Analysis

Before that, we asked the students to solve problem 3 without suggesting the method mentioned above

(introduced in grade 10 physics textbook). The results showed that only 4 students (12.5%) know how to calculate work of changing force by integral. Most students were unable to have any solving strategy even though they can calculate work if the force is constant.

We asked another 15 students to solve this problem, but they were notified about the method in grade 10 physics textbook. In Calculus 1, students were introduced to the definition of integral as the limit of Riemann sum derived from calculating the area under the curve. We want to test whether the students can recognize the application of integral in this physics context.

The results showed that 80% (12/15) students were able to see that the mathematical tool to calculate work for changing force accurately is integral. 83% (10/12) of those students can explain the reasons behind using integral. They argued that “*when dividing work into smaller segments and calculating the sum of that, we use integral*”. Other students could also see the similarities between calculating work and the area under the curve. A few recognized that calculating work can be accurate by switching to limit and integral is the result of that limit.

*Conclusion from experiment:* In order to teach integral integrated with physics, teachers must ensure that learners can use integral to solve physics problems. If so, can pre-service teachers use integral in these problems? The results of our experiment showed that students face difficulties in applying mathematical knowledge in physics contexts. Furthermore, one noticeable thing is that the difficulties they had did not stem from the lack of mathematical or physics knowledge. In problem 1, students had already known that the distance can be found by integrating velocity function as introduced in grade 12 mathematics textbook. These students also knew the geometrical meaning of integral which is the area under the curve. However, when introduced to a situation where the velocity function is represented by a graph instead of being expressed as an algebraic form, most students in our experiment was unable to link integral, the distance and the area under the curve together and reach a suitable solution. In problem 2, despite having adequate knowledge in both mathematics and physics such as the Newton-Leibniz formula or the fact that acceleration is the derivative of velocity, most students failed to incorporate that into the problem which required them to calculate the change in velocity. So, if the difficulties the students face do not stem from the lack of knowledge, then where does it stem from? We suggested that the

problem came from the isolated knowledge that students have. When a student learns a mathematical concept, this concept is taught separately from realistic contexts and science contexts in which the concept is applied. That might be the reason why students could not connect what they have learned to solve the problem despite having knowledge from both fields. The conclusion we made aligned with what Frykholm and Glasson (2005) said. They suggested that teaching integrated mathematics and science needed to be put in contexts that could improve the links and relationships between mathematics and science knowledge.

On the other hand, the results from problem 3 opened up new possibilities for physics being a tool that can help learners understand mathematical concepts better. One of the most important ideas of the meaning of integral is in the Riemann sum. The “divide-sum-limit” method is applied not only to solve math problems in classrooms, but it can also be used to solve other problems outside of mathematical contexts. Students in universities are taught that integral is the limit of Riemann sum but separating this from its various application, therefore can restrict students’ understanding of the integral concept. Introducing students to a situation which requires them to calculate approximately the work done by a changing force showed them the relationship between this approximate method with the Riemann sum in the notion of integral. From that, they could also see that integral can accurately calculate the work done by a changing force and even for many other problems.

#### 4. DISCUSSION AND CONCLUSION

From an integrated standpoint, current mathematics program in high school and in the training of mathematics pre-service teachers do not connect satisfactorily mathematical knowledge with other sciences. Traditional calculus textbooks usually present mathematical knowledge in its pure form and heavily rely on theory without emphasizing applicable contexts in real life and science. This may make students not recognize that their knowledge can be utilized in situations outside the boundary of mathematics. This lack of connections limits their knowledge and prevents them from allowing learners to fully understand mathematics.

We agree with the proposals made by Barnett and Hodson (2001), Frykholm and Glasson (2005) that pre-service teachers need to be equipped with “contextual knowledge” in order to teach integrated mathematics and science. Regarding mathematical concepts, we understand “contextual knowledge” is the understanding of the context that is associated with the development of

the concept or in situations where the concept is a solving tool. In order to prepare pre-service teachers with it, we suggested that the current program should introduce more science contexts that require learners to use mathematical tools. Science can provide contexts and motivations that can enrich the meaning of mathematical concepts for students and enable them to connect with other disciplines. From the experiment, presenting mathematical concepts such as integral, the current program can omit the traditional approach like the area of a trapezoid and approach it from a physics context such as calculating distance or work of a force. We think that the training program at the University of Education should offer more modules related to integrated mathematics and science. These modules can provide students with knowledge and interdisciplinary skills connecting mathematics and science and real-life cases. An assertive step in reforming the training program in the University of Education with the orientation of integrated mathematics and science is a vital part in the premise of reforming education in the future.

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