DESIGNING SITUATIONS IN TEACHING MATHEMATICS BASED ON RME'S CORE PRINCIPLES

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Abstract: This article aims at analyzing the core teaching principles of RME (theory of Realistic Mathematics Education) and constructing teaching situations in accordance with these principles to clarify the meaning of using real context in teaching mathematics.

Keywords: RME, teaching principles, teaching situations.

1. INTRODUCTION

Innovating Mathematics teaching methods to train people to meet the social requirements of the 21st century is an urgent need for the whole education sector in Vietnam and in the world. In each country and region, there are many specific projects or programs for reforming education in general and mathematics education in particular. In the field of math education, there are many educational programs and assessment programs such as NCTT, TIMSS, PISA, RME, etc. Studying these programs, we find out that most of them emphasize on the connection between mathematical practice and knowledge, in which student's activities become the center and are considered as a substitute for the teacher's position.

RME is the theory of mathematics education that associated with reality was established in the Netherlands in the 70s of the last century, based on the idea of a link between Freudenthal's mathematical knowledge and its practical objects. This theory contributes to the development of the mathematical education in not only the Netherlands, but also in many countries such as Britain, USA, Germany, Singapore, Japan, Australia, etc., and most of these countries have achieved some certain results [1].

In Vietnam, a comprehensive education program has just been launched and it emphasizes the skills that students need to gain, such as problem-solving skills, communication skills and reasoning skills, etc. In this program, there will be a significant change in education, particularly in the mathematics education in the coming years. In our research, based on references [2], [3], [4], [5], we realize that the idea of RME theory is well suited to innovating Math education in high school today. Basing on this theory, we will develop teaching situations linked to reality in order to overcome the difficulties that teachers face with when identifying situations, and help students interact with situations to connect mathematics with real world.

In this article, we will summarize the principles of RME Mathematics Teaching and analyze the development of a mathematical teaching situation in accordance with these principles. Then it will provide evidence to prove that it is possible to use real-life situations to discover, consolidate mathematical knowledge and use mathematical tools to find answers to arising problems in reality.

2. CONTENT

2.1. About RME

RME is an acronym for the English phrase "Theory of Realistic Mathematics Education" which is translated as "mathematics education linked to reality". This is the theory developed by Freudenthal and his colleagues in the 1970s at the IOWO Mathematics Education Development Institute in the Netherlands. Based on Freudenthal's statement that people should learn math as an activity, the core principle of RME is that the "formal" mathematical knowledge can be described from the children's mind. Therefore, students should contribute to the teaching/learning process as much as possible and wherever they can. Thus, according to him, mathematics must be linked to reality, close to the student's experience and relevant to society; studying math does not have to focus on receiving the knowledge that is available, it is the process of establishing and solving problems that arise from reality or mathematics itself to create knowledge. With RME, students, not teachers, are the main players in this process of knowledge creation. Students should be facilitated to promote their dynamism, creativity instead of absorbing the theories or the concept presented [6]. After the birth of RME, he contributed greatly to the development of mathematics in the Netherlands, whereby the Netherlands has maintained a high position in international comparisons of mathematical achievement and satisfied results at TIMSS, PISA [1].

In the US, RME has been adopted in the project "Mathematics in Context" with the purpose of

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developing mathematics for high schools in America. The project is funded by the National Science Foundation and is conducted by the Center for Scientific Research in Mathematics at the University of Wisconsin-Madison and the Freudenthal Institute at Utrecht University. The philosophy of this curriculum and its development is based on the belief that mathematics as well as any other science is a product of human creativity and social activity [6]. In Germany, the project "Math 2000" was conceived in 1985 at the University of Dortmund which focusses on mathematics education at all levels, including the basic components "mathematical", "exploration", "arguments" and communication basing on the idea of linking Math with practice proposed by of John Dewey, Jean Piaget and Hans Freudenthal [1]. In Norway, RME is also used to develop National Education Programme L97. In L97, the researchers asserted that education must be attached to the observations and experiences of students [1]. Furthermore, RME is also welcomed and used by researchers in England, Spain, South Africa, Japan, Malaysia where the results are very positive [7].

2.2. RME's core teaching principles

Studying the influence of RME on the education in other countries in the world, we have reviewed the findings in some studies [3], [4], [9], [10]. The six core teaching principles of RME are as follows:

2.2.1. Activity principle: According to RME, mathematics is the implementation of mathematical activities. Therefore, in the teaching process, the teacher must address problematic situations, support students to deal with the problems step-by-step through math activities to help them gain mathematics knowledge.

2.2.2. Reality principle: According to RME, learning mathematics is to solve the problems posed in reality to see what value mathematics can bring about. When teaching with RME, teachers should not impose or offer concepts, or formulate directly but teach through mathematical situations or problematic situations to help learners understand the meaning of mathematical knowledge that they need to gain when solving that problem. These situations are not necessarily taken from reality but may be hypothetical, and the learners accept them when they interest the learners.

2.2.3. Level principle: By dealing with arising situations during the teaching process, the teacher will help students acquire different levels of knowledge, step-by-step. When a problem is solved, each student can move to the next level of.

2.2.4. Coherence principle: This is the principle which requires the teachers to integrate many mathematical disciplines such as algebra, geometry, calculus, etc. together. They need to exploit the links between these disciplines to achieve the purpose of teaching, and encourage students to do assignments or solve situations by applying knowledge of multiple disciplines or using the tools of different disciplines.

2.2.5. Interactive principle: According to RME, learning math is a social issue. It is not only an activity of each individual but also of a close-knit team. RME math educators need to facilitate learners to share their experiences, discuss, exchange ideas and collaborate to help each other improve or discover new knowledge.

2.2.6. Guiding Principle: Learning with RME is learning how to recreate mathematical knowledge. This kind of learning is similar to the way in which mathematicians have previously undergone but not the same as the past. Teachers act as facilitators to help students rediscover knowledge, understand more deeply and memorize what they have learned longer.

Designing the teaching situation of mathematics under RME core principles: Although RME has been well accepted and used worldwide, recent research has shown that there is a great gap between the knowledge learned in school and the knowledge required for practical experience [5]. In this article, we develop teaching situations to connect practical knowledge with students' mathematical knowledge.

Example 1: A case is set up to teach students in the Knowledge Assurance section of the 12th grade textbook using the core principles of RME.

SURVIVAL PEOPLE: "*A carpenter had a block of triangular pyramid wood with volume* $1m^3$*. In order to make an art object, the first time, he took the midpoint of the side edge of the block, marked and conducted the saw by the cut face created by the marked point. He received a piece of pyramid and a triangular truncated pyramid of wood block with the sides corresponding to ½ edge of the original block. For the second time, he marked the midpoint of the edges of the pyramid block obtained after the first and conducted the saw through the cross section through the marked points. He again received a pyramid and a truncated pyramid with corresponding sides equal to half the sides of the pyramid block at the second. He continued the process until it was over. How many volumes of the total pyramid blocks were cut off from the original block?*".

Now, we analysis the situation according to RME's principles: First, there is a practical situation that needs to be addressed. In order to ensure the practical principle, this situation needs to be mathematized. Then the teacher can help students model it into the popular problem as follows: *"For the SABC pyramid, call, $A_1, B_1, C_1, A_2, B_2, C_2, ..., A_k, B_k, C_k, ...$, respectively, the midpoint of SA, SB, SC, SA_1, SB_1, SC_1 ,..., $SA_{k-1}, SB_{k-1}, SC_{k-1}$, ...,. Calculate the total volume of the pyramids $SA_1B_1C_1 + SA_2B_2C_2 + ... + SA_kB_kC_k$ ".*

From the initial real situation, the teacher asked students to use the geometric knowledge to solve the problem. (Ensure RME's practical principles).

To help students solve the situations, the teacher asks the students through discussion, group work and undertakes the following activities: Ensuring activity principles and guiding principles.

Step 1: Cut the $SA_1B_1C_1$ pyramid from the original SABC pyramid. Then, the teacher asks students to answer the following questions:

Question 1: Calculate the volume of the $SA_1B_1C_1$ pyramid according to the volume V of the initial pyramid? (*Figure 1*)

Hint: If V is the volume of the SABC pyramid, V_1 is the volume of the $SA_1B_1C_1$ pyramid, then we have

$$\frac{V_1}{V} = \frac{SA_1}{SA}\frac{SB_1}{SB}\frac{SC_1}{SC} = \frac{1}{8}.$$

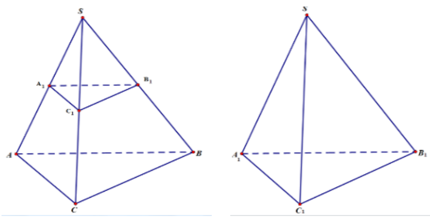

Figure 1

Question 2: Name N as the midpoint of CB, M as the midpoint of AB. Compare the volume of $SA_1B_1C_1$ pyramids with the volume of BB_1MN pyramids? (*Figure 2*).

Suggested answer:

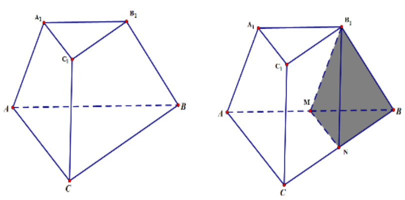

Figure 2

Because $SB_1 = B_1B$ (B_1 is the midpoint of *SB*); $BN = C_1B_1$ (C_1B_1 is the median of the *SBC* triangle); $BM = B_1A_1$ (B_1A_1 is the median of the *SAB* triangle); $B_1M = SA_1$ (B_1M is the median of the *ASB* triangle); $B_1N = SC_1$ (B_1N is to the median of the *BSC* triangle); $MN = A_1C_1$ (equal $\frac{1}{2}AC$). So, the volume of $SA_1B_1C_1$ pyramid is equal to the volume of BB_1MN pyramid.

Question 3: Compare the volume of the pyramid which has been cut off with the volume of remaining truncated pyramid?

Hint: V_{cut1} is called the volume of the remaining pyramid block after cutting off the $SA_1B_1C_1$ pyramid block. Then $V_{cut1} = V - V_1$ and $V_{CC_1NM} = V_1$, so volume of the $SA_1B_1C_1$ pyramid occupying $\frac{1}{7}$ the volume of the rest of frustum, ie $V_1 = \frac{1}{7}V_{cut1} = \frac{1}{7}(V - V_1)$.

After completing step 1, the teacher may ask groups of students to calculate the volume of the pyramids after cutting the second and third time in succession ... Then compare the results of the groups together in step-by-step (Ensure interactive principle).

Step 2: Call A_2, B_2, C_2 respectively is the midpoint of SA_1, SB_1, SC_1 and V_2 is the volume of the $SA_2B_2C_2$ pyramid cut (figure 2). To fulfill the requirements as in step 1 the student also finds the ratio of the volume of the $SA_2B_2C_2$ pyramid and the rest of the pyramid in step 2 is $V_2 = \frac{1}{7}V_{cut2} = \frac{1}{7}(V_1 - V_2)$.

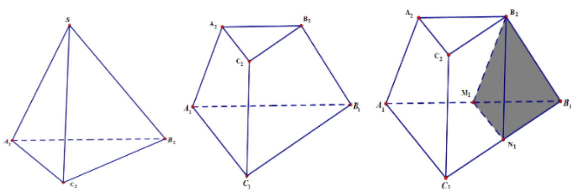

Figure 3

Completely similar to step k we have $V_k = \frac{1}{7}(V_{k-1} - V_k)$ (*Figure 4*).

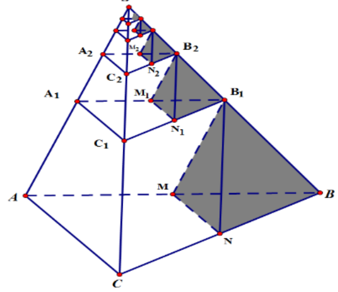

Figure 4

So, the total volume to find is $V_1 + V_2 + ... + V_k =$

$$=\frac{1}{7}(V-V_1)+...+\frac{1}{7}(V_{k-1}-V_k)=\frac{1}{7}(V-V_k).$$

Because the process of cutting off the pyramid is carried out until it can no longer be cut, geometric visualization can be seen through the k steps of V_k volume prediction is zero when k to infinity. So, the total volume needs to be found by $1/7$ the volume of the initial pyramid.

However, this is just a prediction. Teachers can guide students to reexamine the algebraic tools (ensuring the principle of coherence between geometry and algebra), and use old knowledge as the formula for the sum of the infinite multiplier (grade 11) as follows:

The ratio of the volume of the pyramid cut the first time with the initial one is: $\frac{V_1}{V}=\frac{1}{8}=\frac{1}{2^3}$. The ratio of the volume of the pyramid cut the second time with the rest of the first one is: $\frac{V_2}{V_1}=\frac{1}{8}=\frac{1}{2^3}$.

Therefore, the ratio of the volume of the pyramid cut the second with the initial one is

$$\frac{V_2}{V}=\frac{V_2}{V_1}.\frac{V_1}{V}=\frac{1}{2^3}.\frac{1}{2^3}=\frac{1}{2^6}.$$

Similarly, the ratio of the volume of the pyramid cut the third with the initial one is:

$$\frac{V_3}{V}=\frac{V_3}{V_2}.\frac{V_2}{V_1}.\frac{V_1}{V}=\frac{1}{2^3}.\frac{1}{2^3}.\frac{1}{2^3}=\frac{1}{2^9}.$$

And the ratio of the volume of the pyramid cut the k^{th} with the initial one is:

$$\frac{V_k}{V}=\frac{V_k}{V_{k-1}}.\frac{V_{k-1}}{V_{k-2}}...\frac{V_2}{V_1}\frac{V_1}{V}=\frac{1}{2^3}.\frac{1}{2^3}...\frac{1}{2^3}=\frac{1}{2^{3k}}.$$

Thus, the ratio of the volume of the total of the pyramid cut with the initial one is equal to the sum of the infinite multiplier:

$$\frac{V_1+V_2+...+V_k}{V}=\frac{1}{2^3}+...+\frac{1}{2^{3k}}=\frac{1}{2^3}\frac{1-(\frac{1}{2^3})^k}{1-\frac{1}{2^3}}=\frac{1}{7}.$$

We receive the exact result which is the same as algebraic tool in geometric visually: The total volume of the pyramid needs to be exactly equal to $1/7$ the volume of the initial pyramid.

Thus, solving this situation allows students to understand the meaning of mathematical knowledge (multiplication, volume, etc.) in everyday life. This also helps students understand one of the meanings of linking mathematical knowledge to reality and illuminating the practical origins of mathematical knowledge.

Example 2: The situation is built up in the "Extremes of Functions" (12th grade textbooks) to motivate students and stimulate students to seek answers, thereby forming learning needs.

BUY DESK FOR WORKPLACE: *"In the middle of a company A's conference room with a height of 3,4m, a ceiling fan was fitted by the constructing unit. The owner wants to buy a round table with a height of 1m, in the middle of the meeting room. What is radius of a round table he should buy so that when ceiling fan can cool everyone during the meeting?".*

Now, we analyze the situation according to RME's principles: This is a practical situation for students to calculate the radius of a circular table. In order to solve this practical problem, students need to model it into pure mathematical problem (Ensure the principle of reality).

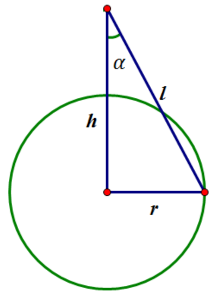

Figure 4

Firstly, we should ignore the elements of air resistance, material, etc., and assume that the fan is a point, and the round table has a radius r, cool wind from the fan out like straight lines of length l. Height from the fan to the table is h. Constant fan speed constant with formula is $\mu=k\frac{\sin\alpha}{l^2}$ ((where k is the fan-dependent ratio, α is the angle between the wind and the edge of the table, l is the length of the breeze). The problem of finding r to μ be the greatest).

Teachers use level and guiding principles to guide student to work in groups (activity, interaction principles) to solve the problem step-by-step through the following questions:

Question 1: What does the radius of the table depend on?

Hint: Using the Pythagorean theorem, we see the length of the breeze is $l^2=(3,4-1)^2+r^2$. So, the radius of the table to look depends on the length of the breeze.

Question 2: What is the relationship between the cool tilt angle and the radius of the table? We have $\sin\alpha = \frac{2,4}{l} = \frac{2,4}{\sqrt{r^2+(2,4)^2}}$.

Question 3: Find the relationship between coolness and radius?

Hint: For assuming cooler intensity $\mu = k\frac{\sin\alpha}{l^2}$ so we have $\mu = k\frac{\sin\alpha}{l^2} = k\frac{2,4}{\left(\sqrt{r^2+(2,4)^2}\right)^3}$.

Question 4: When is the maximum coolness? How to evaluate?

Hint: The coolness is maximum when the junction is maximum for a particular *r*. Problem refers to the survey of the maximum value of the function

$$f(r) = \frac{2,4}{\left(\sqrt{r^2+(2,4)^2}\right)^3}.$$

We have

r	0		$2,4.\sqrt{2}$		$+\infty$
$f'(r)$		+	0	−	
$f(r)$		↗		↘	

From the variation table, we can infer to buy a desk with a radius $2,4.\sqrt{2}\,(m)$.

Thus, by solving this situation, students will firstly understand the role of mathematical knowledge in daily life (using mathematical knowledge to choose the best solution, to make wise decisions in everyday activities) which later stimulates their interest in learning mathematics.

3. CONCLUSIONS

We have presented and analyzed the scenarios designed to ensure RME's six core teaching principles (activity, reality, level, coherence, interactive, guiding). Designing such a teaching situation is often difficult and not all the teachers can do because it requires time investment, practice, and experience, especially the need to be aware of real events through the "mathematical eye" and to link mathematical knowledge to practical situations. However, not all contents in Math textbooks in high schools contains knowledge that is easy to construct such situations. In fact, it requires teachers to study the content of high school mathematics selectively to build practical situations. At the same time, teachers may have to spend a lot of time researching, exploring and creating practical situations that are typical in their lessons to help students understand the meaning and role of mathematics in daily life, thus stimulating the passion for mathematics learning in students. For further study, we will carry out experimental research to compare the learning outcomes between groups of students learning with traditional method and those with RME method to draw pedagogical implications.

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