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DEVELOPING GEOMETRIC THINKING FOR CHILDREN IN EARLY CHILDHOOD EDUCATION THROUGH GEOMETRIC ACTIVITIES

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Abstract: Developing geometric thinking is one of the goals to teach mathematics for young children when they learn by doing and playing. This article proposes the concept of geometric thinking and the role of geometric activities in developing geometric thinking. The study suggests some methods of developing geometric thinking for young children.

Keywords: Geometric thinking, geometric activity, early childhood education.

1. INTRODUCTION

Innovation project of textbook content emphasized on the children's development of capacity and thinking during their early childhood education (children less than eight years old), in which they have plenty of opportunity to solve the problem by themselves, learn how to solve the problem with other people, and apply the knowledge to solve the situations in practical life for individuals and communities. By doing that, they learn how to contribute to "reduction". Developing children's thinking through the teacher established and organized activities is regarded as the guiding ideology in the innovation of teaching methodology. According to A. N. Leonchev, the discovery of the common structure between the two load types of inside activity and outside activity is one of the most important discoveries of modern psychological science: Inside outside activities have the same structure. Inside activity comes from outside activity and is formed from outside activity. Thinking as inside activity is formed through the human's process of working.

Geometry teaching is a favorable condition for development of thinking for learners - especially the quality of flexible, independent and creativity thinking in order to develop spatial imagination and intellectual activities such as comparison, analysis, synthesis, and generalization. At the primary level, the formation of geometric elements and geometric quantities also provides children with fundamental knowledge of mathematics and helps them be aware of the world around them and learn mathematics at higher levels in the future. Indeed, the method of using the geometric activities plays an important role in teaching geometric elements and geometric quantities and develops learners' thinking. Through geometric activities, the children can learn in the activities and via the activities, they can develop their self-doing and self-deciding. They also experience practical life which promotes their positiveness, independence, creativeness, and interest in the Math class which all keep them innocent and happy during early school period.

However, organizing geometric activities at school has some limitations. Although the content of early childhood education program now focuses on more geometric activities, but teachers do not always evaluate the level of thinking and awareness of children in geometric activities. The geometric activities also occur scatteredly and disorderly during the process of geometric elements and geometric quantities teaching. The teachers, in fact, have not applied effectively in the Review class or extracurricular activities, especially the 2 sessions - class / one day in elementary school. They also usually recommend or deduct from reference books, so primary school children mainly take time to do homework in class. The teachers also focus on teaching knowledge more than training and developing thinking for children and organize fewer activities for children. The reason is that teachers do not pay proper attention to the role of geometric activities for the development of thinking, the establishment and organization of geometric activities which is time-consuming and they often feel hard and confused when evaluating and controlling children.

2. CONTENT

2.1. Literature Review

Dutch mathematics educators Van Hiele categorized geometric thinking levels from low to high and designed the evaluation tests for children and teaching methods which are applicable to each level and widely used in designing the teaching content (for early and high schools) in many countries in the world

Geometric thinking requires a combination of spatial concepts and symbols, logical thinking and spatial perception. Clements said that even when the children know that a triangle has three sides and three angles, they

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called the "▲" a triangle but they cannot accept the "▼" as a triangle because it "spins down under ". In other words, the visual icon element can "obscure" thinking logic element and language. So the teacher plays an important role in establishing the space, the symbol and cognitive concept [3; pp. 427].

According to Hoffer [4; pp. 12] the geometric thinking is the capacity which teachers need to train children in the teaching of geometry. He gave five groups of necessary skills of geometry thinking:

i) The capacity of visual - images: identifying and observing the characteristics of geometry, reading a map and identifying the different positions.

ii) The capacity of language: using correct terminology and accuracy language when describing the object and spatial relationships.

iii) The capacity of making shape: creating icons of two-dimensional or three-dimensional, drawing similar figures, and drawing symmetric figures.

iv) The capacity of logical thinking: classifying, recognizing criteria to create and test hypotheses and inference, and demonstration.

v) The capacity of applying: applying knowledge learned in practice and solving practical problems by geometry.

The previous studies have revealed the existence of geometric thinking, the factors that affect the capacity of the geometric thinking, and the level of geometric thinking in children. They have studied the role of the geometric activity for the formation and development of geometric thinking. However, the evaluation of the geometric thinking in geometric activity has not been interesting. In the same content of teaching or the same geometric activity, the children show different ways to solve problem, how to think, and how to solve the problem and the different difficulties and "obstacles". Therefore, it is necessary to have initial research focusing on the evaluation of geometric thinking capacity in geometric activities. On the other hand, the construction and organization of geometric activities to promote geometric thinking and differentiate children has not really been focused, diverse, and rich and has not shown the way of development from low level to high level.

2.2. Suggest concept of Geometric thinking

Geometric thinking can be understood as: *"the process including: 1) the perception of space; 2) the implementation of intellectual activities such as comparison, analysis, synthesis, generalization, abstraction symbolized, inference, improvement; 3) the solutions to the problem; and 4) the application".*

It is important for children to do the intellectual to decide if children have geometric thinking or not.

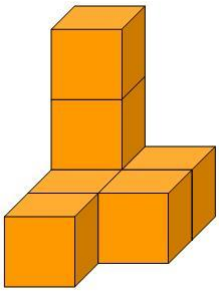
From the above conception, the characteristics of geometric thinking can be described as follows

1) The intellectual activities of geometric thinking is based on the perception of space

In the work of thinking through eyes, Arnheim argues that "*the most important intellectual activities derived directly from our perception of the world, and his view is considered as premium sensory system which covered and formed our cognitive processes."* "Spatial perception itself is not thinking process but is necessary to carry out the geometric thinking, so it will be difficult to solve geometric Mathematics without drawing observation or at least thinking about drawing. Spacial perception is included in the perception of spatial objects and spatial relations and movement combination between the senses (mostly the combination between hand and eye). When teaching the first age at school, the early basis visual is more significant because the child's thinking is mainly visual thinking.

2) Geometric thinking has a close relationship with space imagination

- Geometric thinking is the condition for children to carry out space imagination



Picture 1.1

For example: "How many cubes are there in picture 1.1?" This exercise requires children to use space imagination in solving problems. However, that imagination is only formed in the process of observation, puzzle, anasis, and comparison of cubes in practice.

If a problem that cannot be solved by geometric thinking, spatial imagination can help predict results such as "solving" method for the situation. Moreover it creates a destination for thinking towards, or in other words, it orients to geometric thinking. Thus the difficulties of geometric thinking is a premise for spatial imagination, and spatial imagination becomes the destination for geometric thinking to develop.

3) Language is a part of geometric thinking

4) The children can apply in practice which is a base to access geometric thinking

The ability to apply in practice is an important criteria to determine children's geometric thinking capacity. They began to approach the specific objects and relations, practice intellectual activities and return to the application with the specific objects around them. The teaching of geometric elements and geometric quantities not only helps children learn things of objects or spatial relations, but also requires children to practice.

On the base of the study of the characteristics of thinking in early child, the study of level of geometric thinking, according to Van Hiele [3; pp. 424], can be understood: *"geometric thinking at this age is the process including: 1) the perception of space; 2) the implementation of intellectual activities such as comparison, analysis, synthesis, 3) the solutions to the problems and 4) the application. "*

2.3. The role of geometric activities in developing geometric thinking

Geometric activity is the activity which is based on geometric models. In order to enhance the teaching practice, the establishment of geometric activities is necessary.

Because the geometric activity is centered and is the basis of geometric thinking for children, the geometric activities take place very early in children through drawing activities, puzzles, coloring, and squeezing in kindergarten and continue to develop in elementary school. We found that the study of "vertical" continuously develop and the connection between the level of education is necessary. Thus the teachers found the way to develop geometric thinking and geometric activity in children and appropriate teaching methods, find the way to solve problems during geometry activities, set up knowledge-tectonic situations, and innovate the assessment process to reduce content in textbooks.

Performance of geometric thinking capacity of early child during geometric activities

In the process of geometric activities for children, there can be proposal of evaluation of geometric thinking through the followings:

1) The capacity of performing intellectual activities (comparison, analysis, and synthesis) is reflected on:

- Means which children used in activity: Which means and materials do children choose? How to use them?

- Which difficulties, "obstacles" do children have in the thinking process?

- The result of activity: What is the level of completion?

2) The capacity of the language:

- How do the children use the terms and common language to describe objects and spatial relationships?

- How do the children express the way to solve problems with other people?

3) Capacity of applying: What can children do in practice? Can the answer and explain the specific objects and spatial relationships around them?

In addition, each activity has its own characteristics that can require more performances of intellectual activities such as: the activity to classify shapes, which criteria can the children use to classify? Do children use one or many criteria for classification?

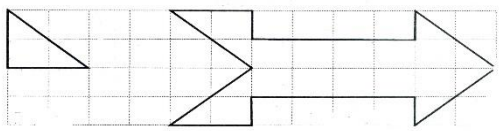
2.4. Suggestion to develop young children's geometric thinking

2.4.1. Method 1: Assessing the capacity of geometric thinking of young children in geometric activities

The current level of geometric thinking capacity of the children will affect the way they act or react to solve the problem, so it is important to decide what we can learn from these activities. Recognizing the level of geometric thinking helps teachers understand how the children will react or act, understand better why they can do these activities but not the others, and why some children have difficulty in this activity while the other children do not. The classification of geometric thinking through activities also helps teachers recognize the "obstacles" that children have to overcome the process of geometric activities and achieve the goals of Geometry teaching. The performances given are suggestions to help the teacher to observe the children in the process of geometric activities, and prove the level of children's geometric thinking capacity, and suggest teachers the ways to organize other geometric activities which is suitable with the level of children's geometric thinking.

Current teaching programs are usually designed based on the logic of content development. Therefore,, each content of teaching of geometric objects or geometric relationships in the program can be designed activities for children to go through the levels of geometric thinking. For example:

Give 8 equal triangles: Please put into the shape of a fish as the sample. The teacher organizes geometric activities including 8 triangles as the fish-shaped figure and then ask them to pave.



Picture 1.2

Then we can divide the levels of geometric thinking capacity.

Level 1: Practice in the "trial and error" process.

Level 2: Flat paving and tell the way to move the shape.

Level 3: Visualize before moving triangle: "How many triangles do you need to create the fishtail? How many triangles do you need to create the body of fish?" "How do you create fishtail?"

- The hierarchy of level of geometric thinking capacity does not mean that the children have to achieve every level in all the activities. Depending on the content of Mathematics teaching which can be achieved sooner or later, in the general situation children can overcome the steps of thinking. For example: In the introduction to rectangular parallelepiped in the program of Grade 5, when teacher requires children to draw one given rectangular parallelepiped, children can draw 1 rectangle parallelepiped at level 1, two "consecutive" rectangular parallelepipeds at level 2; or a performance shape from their position; or a rectangular parallelepiped from different observation positions at level 3.

The followings are some suggestions to teaching methods:

- Step 1: Prepare geometric activities;

- Step 2: Prepare observation sheet, analyze and assess children;

- Step 3: Carry out activities, observe and record the performance of children's geometric thinking capacity (preferably with the help of camera and photo camera);

- Step 4: Assess the level of geometric thinking capacity in children based on the results received in Step 3 to make comparison with Table geometric thinking capacity in this activity; then make conclusion.

2.4.2. Method 2: Forming the way to solve the problem in some geometric activities to establish and organize geometric activities

The author built activities to solve problems and organize activities based on the questions to create problems:

The way to solve the problem 1: When creating shape (drawing, folding, etc.), we need to think about the shape of the entire object and the relationship of parts as well as the relationship of parts with the whole objects.

The way to solve the problem is creating shape to keep the original shape, it means that they "congruent" with the original object and the children need to:

- "see" the constituent parts of whole objects;

- create each component part;

- connect them in the correct relationship with each other and in relationship with the entire object.

The way to solve problem 2: An expansion of shape must have the component shapes that are similar to cubes and components must be placed in a certain way with each other.

Through practical work which focuses on the components of a cube and the way to "match" them with each other, children need to understand that the expansion of any shapes will need :

- figures which have the appropriate shape and size;

- full of sides;

- the correct position with each other.

The way to solve the problem 3: To demonstrate the performance shape of the cube, we need to combine what we can actually observe with something we imagine.

There are several ways to approach the performance of shapes on a plane, most adults can recognize that the shape looks like a cube. For them, the cube has "turned out" and looks cubic. However, children may not realize that it is a cube, they may "see" it as a "hexagonal" and there are some lines which crossed together in the figure. When being asked to draw again, they could draw a picture included in three "diamonds". Therefore, children must learn to read the performance in the plane with its limitation to "float" these figures.

Mathematics is equipped with a number of rules to represent cube which are widely used and the children need to learn the rules to explain and re-create them. Children must learn to know that:

- they can show what can be seen from a specific position and ignore something that we know they are there but cannot seen (we do not draw all four wheels if we can only see three wheels).

- they can "distort" the shape and size to make the drawings look more realistic, for example, drawing wheel as ellipse and display the further objects are, the smaller they are as the perspective drawings.

- they can draw differently from the reality to create a emphasized point on some characteristics, as the

oblique and edge drawings.

The way to solve the problem 4: We can move the object around in space by moving and rotating the image. The action does not change the size or shape.

The ways to solve the problem:

- looking at the entire figure which is needed to be paved and constituent parts;

- analysis the figure that cannot be paved and the figure can be paved based on shape features;

- the children use the rotate action and movement action to lock into position to pave.

The way to solve the problem 5: The object can have the same characteristics and different characteristics, when being classified, we arrange the objects in the same group which they have the same characteristics.

Classification is the common activity in mathematics, such as classification of even and odd numbers in series of natural numbers, or classification, arrangement of the data is a common activity in content of descriptive statistics. The ways to solve this problem in the teaching of geometric elements and geometric quantities is when are classifying we help children orient the criterias for classification and children have to analyze, synthesize, and compare the characteristics of the geometric objects.

The way to solve problems helps teachers with an general overview of the organization of geometric activities, with a clear goal in the establishment and organization of geometric activities. Each geometric activity not only improves the learnt symbols and concepts but also contributes to the way of solving problem for a Mathematics class and class activities. We also need to understand that in order to formulate ways to solve the problem, the children also have step-by-step the intellectual activities such as comparison and analysis - synthesis, and generalization. Thus the orientation of the main ways to solve the problem is to develop the intellectual activities and contribute to the development of geometric thinking in children. The statement used to solve the problem in the form of clause also helps teachers remember, carry out and make deeper and more detail mathematical knowledge of pre-school and primary school teachers.

2.4.3. Method 3: Increasing the knowledge used into practice during the process of geometric activities to help children answer and explain the basis of many phenomena in the surrounding environment

2.4.3.1. The exploration of the object shape: The abiotic and biotic features have specific shape and structure which is usually related to their function and adaptation. People also use the shapes and transformations as a basis for design. In the first years in school, we should focus on exploring (both free and organized methods) environment around children and the world of objects. The nature of this method is based on an exploration of the geometric shapes to help children begin to explain the function of the shape of objects in the environment around them. Why and when is a geometry used that way? For example: "Why don't we use a square for a wheel?" "Which characteristics of the sphere makes it useful to the ball game?" "How can we make the walls look more solid? " This content is reflected on what they can do in practice when they learn geometric elements (in the context of the geometry).

2.4.3.2. The measurement in practice : The measurement in education program takes much time with additional support from the content of Arithmetic. Within the scope of this method we propose to enhance the measurement of geometric quantities in practice. The geometric quantities in practice may be the length, width and height of an object. For the length of an object or the area of a flat, we need to develop an understanding before they learn "deeper" in the next grade. The goal of this section is to orient children to understand the measurement of a geometric quantity and to consider how many parts of the unit they need and then compare two quantities using the unit of measurement to estimate or use formula. This process helps build and organize activities for young learners.

2.4.3.3. The oriented activities in space: Every object in space has position determined by the origin and depended by the observer's position. In the process of teaching to encourage children to use the words that point out the relative position in space: "in", "out", "left", "right", "over", "under" , "in front of", "behind", and the direction of movement such as "straight," behind ", etc., the children can say "it is next to the phone, "the right bank of the river," the first turn left and then turn right "and" there". Children should be encouraged to develop the daily language of location and spatial arrangement which includes the use of nets.

Children will need a lot of experience in arranging and rearranging the familiar objects in freedom and under the direction, and need opportunities to try their language. Early ideas for the question "Where is it?" should be expanded to contain information about the spatial orientation such as "across from the door", "on the left", "in the north of the door". They also need to getting familiar with the pair of orientation "top",

"bottom", "upon clockwise", "counter clockwise" and movement "forward", "back", "go round", "turn around" and develop words of space icons.

The grids and coordinates provide many mathematical properties to represent the position and orientation. The essential idea is that we can use number (usually a pair of number) to represent an object in somewhere. Child's experience of the location of objects on a grid is mostly square and crossed row and column.

In early school, orientation activity in space is a required content and they can locate themselves against others or the relative location of objects to each other.

3. CONCLUSION

Methods of geometric elements and geometric quantities teaching for children in early school should start with the real world of children. Situation to make children familiar with spatial objects and spatial relationships should be used. When children are experienced in this situation, they must be able to reflect them. Geometry does not start with definitions and theorems. Tt started when the children orient themselves in daily surroundings and are familiar with physical environment. It will ultimately helps children have more experience in the development of the definition and theorem.

The establishment and organization of geometric activities help connect the knowledge which children know and real things in life help children consolidate their knowledge, deepen their knowledge and make them aware of the world around them. Organizing activities contributes to develop children's capacity of mathematics in general and children's capacity of geometric thinking in particular, to enhance the application of mathematics in practice, and to assess what children can do in practice more than what they know when learning Mathematics. Children use the knowledge which they have just learned to deal with the situations in the close and practical life for individuals, help them answer, and explain daily phenomena. The activities also help children be interested in math class, but not be confined to sitting in the classroom. They also develop children's feelings, their emotional and social relationships or physical education.

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