

THE TREND OF USING HAND-HELD CALCULATORS FOR TEACHING AND LEARNING MATHEMATICS AT SCHOOL

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Abstract: Hand-held calculators have been introduced to schools in many countries around the world for more than 40 years. Many studies have shown that hand-held calculators with compact size and installed mathematical functions have considerably benefited the teaching of Mathematics. This article discusses the trend of using hand-held calculators for teaching Mathematics in schools in Vietnam.

Keywords: Hand-held calculators, teaching Mathematics, students.

1. INTRODUCTION

The objectives of Mathematics education in the General Education Programs include forming and developing student's core qualities, general competence and mathematical competence. In particular, the development of mathematical competence require students to utilize thinking and reasoning skills; to calculate, estimate and use calculating and measuring tools. In addition, high school students need the ability to effectively use hand-held calculators and some calculation/statistical softwares in their study and life [1].

The use of hand-held calculators in the teaching of Mathematics in schools is a necessity and should be objectively and scientifically considered for inclusion in the education program. Based on the studies on the trend of using hand-held calculators in some countries, the benefits, difficulties and challenges of using hand-held calculators for Mathematics teaching, this article recommends some measures to promote the efficiency of this calculating tool for teaching Mathematics in schools in Vietnam.

2. CONTENT

2.1. A brief history of hand-held calculators

- *Before 1950: Mechanical calculators - the precursor of electronic calculators.* The earliest calculating machine was the Abacus used by the Sumerians and Egyptians in 2000 BC. In 1642 of the Renaissance, the mathematical genius Blaise Pascal (1623-1662) invented the mechanical calculator, the first device to perform basic calculations without using human intellect. This device could directly perform additions, subtractions, multiplications and divisions. Then some other improved mechanical

calculation tools were introduced, but due to technological factors, the calculation tools in this phase could not perform various calculations, and they remained weighty and massive.

- *From 1950 to 1970: The introduction of electronic calculators.* The large-sized-calculators using vacuum tubes and then transistors to solve logic algorithms first appeared in the 1940s and 1950s. This technology was a great step forward for the invention of electronic calculators. In 1957, Casio Computer Co., Japan released the 14-A model. This was a comprehensive electronic calculator with the world's first compact design.

- *From 1970 to 1980: Pocket-size compact calculators.* The introduction of microchips and electronic chips was an efficient solution. Manufacturers managed to create microchip boards with small built-in transistors that allow creating smaller, less power-consuming calculators. The first pocket-size calculator was released in 1971, which was a LE-120A manufactured by Basicom of Japan.

- *From the mid-1980s to present.* The rapid development of science and technology allowed manufacturers to produce cheap scientific hand-held calculators. Conventional hand-held calculators with basic calculating functions only cost a few dollars, but still allowed performing accurate and reliable calculations. In 1985, the first calculator capable of graphing a given function was the Casio FX-7000G. Entering the 21st century, the gap between a hand-held calculator and a laptop has been significantly narrowed. Many modern scientific hand-held calculators can calculate differential and integral functions, solve differential equations, process

character sequences, and run personal data management software. Hand-held calculators are also equipped with wireless connection and an infrared port to communicate with computers or other scientific calculation tools.

In short, over the past 400 years, since the Pascal's invention of primitive mechanical calculating machine, a series of technological improvements as well as computing program and hardware upgrades have been carried out. These days, we have pocket calculators which weigh less than 200 grams.

2.2. The trend of using hand-held calculators for teaching in some countries around the world and in Vietnam

The trend of using hand-held calculators for teaching Mathematics has been increasingly developed in many countries. In general, hand-held calculators have been vigorously and commonly used in schools in some countries around the world. Along with the development of science and technology, graphing calculators have also been employed. In Vietnam, hand-held calculators were introduced to schools in 2000.

- *In Australia:* The use of hand-held calculators by students in learning Mathematics gained popularity in the late 1970s, when scientific calculators were approved for use in final exams. Since the 1980s and in the current national textbooks, students have been allowed to use digital devices, including hand-held calculators, as well as other calculating methods during their study.

- *In the United States:* Hand-held calculators have been used for teaching Mathematics since the 1970s, and have extensively developed today, especially since the introduction of graphing calculators in 1985. In the curriculum of Advanced Placement (AP) Calculus, teachers started applying graphing calculators in 1991, and in May, 1995, the US School Councils allowed using graphing calculators in the exam of Advanced Placement Calculus AB and BC. In the statistical mathematics curriculum, graphing calculators were quickly recognized as an important teaching and learning tool, enabling both data analysis and statistical inference. Graphing calculators have also been the necessary tools for candidates to take AP exams since 1998.

- *In Singapore:* The use of graphing calculators in teaching was first permitted in 2002. Until 2006,

students were allowed to use graphing calculators without the CAS (*Computer Algebra System*) in advanced exams national. However, in these exams, questions were built in such a way that students who did not use graphing calculators would not be disadvantaged.

- *In Malaysia:* In Malaysia, using hand-held calculators for teaching and learning Mathematics has received much attention from the Ministry of Education (MOE). Since mid-2003, MOE has provided teachers and Maths teachers with short-term training courses on hand-held calculators, or workshops as part of teachers' professional development program. This program is expected to continue until the majority of Maths teachers can be familiar with the use of modern calculator technology.

- *In some European countries:* In UK, hand-held calculators are permitted for use with Level A Mathematics; however, they are not compulsory, and exam questions can be solved with or without calculators. *Scotland* allows the use of calculators in Maths exams. In *Finland and Slovenia*, students are not allowed to use calculators for algebraic calculations or with 3D graphing functions in the certification exam. In *Netherlands*, high school students can use graphing calculators in tests and exams in the last three years. Graphing calculators are allowed to be used in tests instead of conventional ones.

- *In Vietnam,* hand-held calculators were introduced in the 1980s. In the primary education program, calculators first appeared in grade 5, with the role of checking the results of calculations as students learn about decimals. However, due to the limited economic condition, teachers and students were less likely to use them. The 1990-2000 general education program hardly mentioned the use of hand-held calculators in Mathematics instruction. In recent years, the Ministry of Education and Training has encouraged the introduction of hand-held calculators to the teaching of Mathematics in schools. In the current general education program, the use of hand-held calculators is regulated to be included in textbooks, indicating that students can practice and use calculators in tests and exams.

Although hand-held calculators have been encouraged to use widely in schools, the use of hand-held calculators in Mathematics instruction is

restricted in China, India, Finland and some underdeveloped countries due to different viewpoints on the influence of using calculators in the teaching of Mathematics (the possibility of reducing the calculation skills other concerns about technical and economic conditions which has not allowed massive usage of calculators in classrooms).

2.3. Benefits of using hand-held calculators to Math teaching at school

Using hand-held calculators for teaching Mathematics is advantageous because of the following characteristics:

- With its compact size and easy mobility (portable, pocket-size), a hand-held calculator makes it easier for students and teachers to use than some other electronic devices which are being used for teaching today.

- Related to calculating ability, today's hand-held calculators are designed with assistance and consultancy from educational specialists with the primary aim to support teaching and learning, so they can basically satisfy the needs of Mathematics teaching to a certain extent.

- Hand-held calculators are more economical technological devices in comparison with other types of calculating tools such as laptops or desktops. It is, therefore, affordable to equip individuals with calculators thanks to their low cost.

Some studies have shown the benefits of hand-held calculators to Maths teaching. Hand-held calculators help students improve knowledge, abilities of solving problems and calculating skills. Robova's study indicated, "*The use of hand-held calculators in the teaching of Mathematics creates new working methods, particularly the ability of predicting and modeling Mathematical problems and graphically supporting for the results obtained by algebraic procedures*" [2]. In the hand-held

calculator environment, teaching and learning activities performed by teachers and students become more active. The study by Hembree and Dessart pointed out, "*Students who use hand-held calculators have better learning attitudes and ability of Maths self-learning than students who do not*" [3]. A study by Dunham and Dick also asserted that "*Hand-held calculators with graphical functions can help students better solve problems, enabling changes in the role of students and teachers, leading to an interactive and discovering learning environment*" [4].

Some recent studies have indicated that using hand-held calculators helps students better visualize problems, explore Mathematics, test hypotheses and find out different solutions to solve problems [5]. Using calculators allows students to independently explore Maths topics and contents, creating an proactive approach to learning, transforming the classroom organization model from students' passive learning to active learning in which students work with their classmates and come up with their own ideas and solutions. Dunham and Dick pointed out that using hand-held calculators [4]: - *enabled students to be more successful in tests*; - *enabled students to have a more flexible approach to solve problems*; - *enabled students to engage in problem solving and remember the problem for longer*; - *enabled students to solve problems that were not accessible by algebraic techniques*.

For example, a teaching situation is designed to solve the following problem with the aid of hand-held calculators:

Problem 1: Consider the consecutive exponents of 2: $2, 2^2, 2^3, \dots, 2^n$.

1) *When these exponents are divided by 5, what types of remainders will be received?*

2) *Could the problem be generalized?*

Teacher's activities	Student's activities	Content																
<i>Activity 1: Calculating to discover the rule of remainders</i>																		
1) Use a hand-held calculator to calculate the remainders of the first 20 exponents of 2 when divided by 5. 2) What comments do you have about the newly found remainders?	1) Use a hand-held calculator to calculate and fill the results in the table. 2) Give comments: The remainders are: 2, 4, 3, 1 and they are repeated cyclically in the order above.	1) Results (the second row indicates the remainders when the exponents of 2 are divided by 5): <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>2^1</td> <td>2^2</td> <td>2^3</td> <td>2^4</td> <td>2^5</td> <td>2^6</td> <td>2^7</td> <td>2^8</td> </tr> <tr> <td>2</td> <td>4</td> <td>3</td> <td>1</td> <td>2</td> <td>4</td> <td>3</td> <td>1</td> </tr> </table>	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2	4	3	1	2	4	3	1
2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8											
2	4	3	1	2	4	3	1											

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2^9	2^{10}	2^{11}	2^{12}	2^{13}	...									
2	4	3	1	2	...									
<i>Activity 2: Proving the results of activity 1</i>														
1) Prove the above results	1) Use arithmetic knowledge to prove: When the exponents of 2 are divided by 5, the remainders are 2, 4, 3, 1	For all natural numbers k, the exponents of 2 will be in the form: $2^{4k}, 2^{4k+1}, 2^{4k+2}, 2^{4k+3}$. With all $k \in \mathbb{N}$, we have: $2^{4k} = (2^4)^k \equiv 1 \pmod{5}$; $2^{4k+1} = 2 \cdot 2^{4k} \equiv 2 \pmod{5}$; $2^{4k+2} = 2^2 \cdot 2^{4k} \equiv 4 \pmod{5}$; $2^{4k+3} = 2^3 \cdot 2^{4k} \equiv 3 \pmod{5}$												
<i>Activity 3: Generalizing the problem</i>														
1) Suggest some similar forms, such as: <i>Problem 2:</i> Find the remainders when exponents of $3, 3^2, 3^3, \dots, 3^n$ are divided by 4? <i>Problem 3:</i> Find the last two digits of 2^{2017} ? 2) Generalize Problem 1?	- It is easy to get results (similar to Problem 1). - Solve Problem 2 in the same way with Problem 1 (but with more calculations).	Here, by observing and comparing, students are able to predict the remainder rule of exponents of 2 when divided by 100 in the twenty- number cycle. - <i>Expected results:</i> General Problem: <i>For any natural numbers a and m which is not 0, the remainders of the division a, a^2, a^3, a^4, \dots for m are repeated cyclically.</i>												
<i>Activity 4: Proving the general problem</i>														
<p><i>Comment:</i> This mathematical problem is mentioned in <i>Mathematics Program of Grade 6</i> (for gifted students). In the above teaching situation, the benefits of using a handheld calculator include:</p> <ul style="list-style-type: none"> - The quickness and accuracy of the calculation of remainders (through the use of hand-held calculators). Manual calculation will take much more time. Instead, using hand-held calculators will help students to have more time to solve mathematical problems rather than spend much time on normal calculations; - From the achieved results, students easily <i>observe, compare, predict and prove</i> the statements. From the similar mathematical problems presented by teachers, students can generalize the problem; - Students are provided with <i>a new approach</i> (using learning aids) and actively involved in problem solving. 														

Despite the benefits of using hand-held calculators in Mathematics teaching in Vietnam, there are still some limitations and challenges in the use of hand-held calculators as follows:

- *Regarding learners:* A number of students do not understand all functions of hand-held calculators, or they are not good at operating hand-held calculators. In addition, the ineffective use of hand-held calculators results in a reduction in students' mathematical learning efficiency.

- *Regarding learning contents:* not all mathematical contents can be supported by hand-held calculators. The designing of Mathematics teaching situations with

the help of hand-held calculators has not been studied and exploited properly by teachers. In testing and assessment, tests have not been effectively designed to prevent students' dependence on hand-held calculators when solving mathematical problems.

- *Regarding technological and scientific factors:* Although hand-held calculators have been significantly improved in terms of hardware and installed mathematical functions, today's common hand-held calculators still expose some limitations. For instance, they are not able to connect to the Internet. Moreover, a hand-held calculator costs much (a normal hand-held calculator costs about \$20

while a hand-held calculator with graphing functions costs about \$150). It is, therefore, difficult for learners to possess.

3. CONCLUSIONS

Today, most countries in the world exploit calculators for the teaching of Mathematics from elementary to tertiary curricula. With the support of hand-held calculators, complicated calculations are easily resolved, and results are clearly defined and demonstrated. Using calculators in the teaching process will help learners construct, develop and explore knowledge, and enhance problem solving ability. Hand-held calculators meet complicated calculating needs; effectively support the access to and transfer of theoretical knowledge; connect theories to practice; help students not only absorb the knowledge of science but also approach modern calculation tools. With the rapid development of science & technology and other technical means, hand-held calculators become more and more useful in teaching Mathematics.

In order to promote the effectiveness of hand-held calculators in accordance with today's development trend and meet the requirements for the reform of general education programs, some implications are recommended as follows:

- *In terms of the syllabus designing:* In Mathematics, it should be compulsory that some mathematical contents be resolved with a hand-held calculator. Teachers should be encouraged to use the hand-held calculator as a tool to approach and explore knowledge, as well as a means of analyzing mathematical situations.

- *For pedagogical and teacher training institutions:* The institutions should design contents that involve the use of hand-held calculators. This is intended to equip learners with knowledge and skills of using learning facilities, and pedagogical skills for using hand-held calculators in teaching.

- *For schools:* In the development of education programs, it is necessary for schools to pay attention to teaching facility supporting and workshop organizing in order to equip teachers with skills of effectively using calculators in the teaching of *Mathematics*.

- *For teachers:* Teachers need to be skillful at operating hand-held calculators in order to help

students effectively use these modern calculation tools in learning Mathematics.

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